Does an Integrated Focus on Fractions and Decimals Improve At-Risk Students' Rational Number Magnitude Performance?

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Abstract

The purpose of this study was to assess whether intervention with an integrated focus on fraction and decimal magnitude provides added value in improving rational number performance over intervention focused exclusively on fractions. We randomly assigned 4^{th} graders with poor whole-number performance to 3 conditions: a business-as-usual control group and 2 variants of a validated fraction magnitude (FM) intervention. One variant of FM intervention included an integrated component on fraction-decimal magnitude (FM+DM); the other included a fraction applications component (FM+FAPP) to more closely mirror the validated FM intervention and to control for intervention time. Cross-classified partially-nested analyses (N=225) provided the basis for 3 conclusions. First, FM intervention improves 4^{th}-graders' fraction understanding and applications. Second, effects of FM intervention, even without a focus on decimals, transfer to decimal number line performance. Third, an intervention component integrating fraction-decimal magnitude does not provide added value over FM intervention on fraction or decimal performance, except on decimal tasks paralleling intervention tasks.

Keywords: math, intervention, fraction, decimal

Does an Integrated Focus on Fractions and Decimals

Improve At-Risk Students' Magnitude Performance?

Research documents that competence with rational numbers, often indexed in the form of fraction magnitude understanding, is important for algebra learning (Booth & Newton, 2012; Booth, Newton, & Twiss-Garrity, 2014; Brown & Quinn, 2007; Empson & Levi, 2011). Rational number skill is also linked to success with more advanced mathematics and post-school employment (Geary Hoard, Nugent, & Bailey, 2012; National Mathematics Advisory Panel, 2008; Siegler et al., 2012). Accordingly, across the middle grades, the school curriculum is designed to gradually expand understanding of number by consolidating principles of whole numbers and rational numbers into a single numerical framework (Siegler, Thompson, & Schneider, 2012).

The instructional shift from whole to rational numbers, which begins with fractions in the U.S. curriculum at grades 3-4, represents a challenge for many students (Durkin & Rittle-Johnson, 2015; Kallai & Tzelgov, 2009; Obersteiner, Van Doorena, Van Hoof, & Verschaffel, 2013; Siegler et al., 2012; Vamvakoussi & Vosniadou, 2010). This is because fractions differ from whole numbers in fundamental ways. For example, a single fraction is expressed via two numerals; fractions with the same numerator become smaller as denominators increase; an infinite number of fractions exists on any segment of the number line; and multiplying and dividing fractions usually produce unexpected increases and decreases.

The rational number developmental hurdle is especially challenging for students who have struggled in the primary grades with whole-number learning. Namkung, Fuchs, and Koziol (2018) estimated that students with below grade-level whole-number knowledge are 32 times more likely than students with adequate grade-level whole-number knowledge to struggle with rational numbers. As Malone and Fuchs (2017) documented, the source of errors often resides with misapplication of whole-number principles to rational numbers: In comparing two fractions, at-risk fourth graders consistently chose the fraction incorporating the single greatest numeral as the greater magnitude, while 65% of errors in ordering three fractions reflected whole-number thinking. Schumacher and Malone (2017) reported similar error types among at-risk fourth graders when adding and subtracting fractions.

The sizable achievement gap in fractions knowledge between students with prior histories of whole-number learning and their not-at-risk classmates (Namkung et al., 2018) indicates the need for intervention to supplement the schools' instructional program. In a series of randomized controlled trials (RCTs), Fuchs and colleagues (Fuchs, Malone et al., 2016; Fuchs, Schumacher et al., 2013, 2014, 2016) tested the effects of a 12-week (36-session) fourth-grade intervention focused on fraction magnitude. At the start of fourth grade, participants were identified as performing below the 35% percentile on a nationally-normed math achievement test. Approximately half of the sample was below the 15th percentile; half between the 16th and 34th percentile.

In each study, findings indicated superior performance for the intervention condition over the control group on fraction magnitude understanding, as indexed on comparing fractions, ordering fractions, and placing fractions on number lines. Effects were also realized on fraction addition and subtraction even though the intervention devotes just 3 of 36 lessons to calculations. Moreover, effects transferred to a distal measure of overall fractions performance: released items from the fourth- and eighth-grade National Assessment of Educational Progress (NAEP). Effect sizes (ESs) ranged from 0.37 to 2.50 depending on study year and outcome, with most in the moderate to large range. Moreover, as Fuchs, Sterba, Fuchs, and Malone (2016) demonstrated, the superior effects of intervention over control were robust across the spectrum of at-risk students' pretest mathematics performance. As reported in the individual studies, the fractions achievement gap between at-risk students and their average-achieving peers was completely erased or dramatically reduced at the end of intervention.

Despite the demonstrable efficacy of such fraction intervention for enhancing the learning of at-risk fourth graders, this series of RCTs did not examine the effects of an instructional focus on decimals. We identified only one previous RCT focused on decimals with low-performing students. Woodward, Baxter, and Robinson (1999) contrasted the effects of conceptual versus procedural decimal instruction in two remedial seventh- and eighth-grade classrooms, which were randomly assigned to the two conditions (there was no control group). Calculations performance marginally favored the procedural condition (p = .06), while interview data revealed stronger understanding in the conceptual condition.

More central to the present study, we identified no prior RCT that assessed the value of *bridging across multiple rational number notations for at-risk learners*. The absence of such a controlled study for at-risk learners is surprising, because Common Core State Standards (CCSS; http://www.corestandards.org/Math/), and the resulting versions of College- and Career-Ready Standards adopted by states nearly universally, specify that fourth graders "use decimal notation from fractions with denominators 10 or 100" and "compare two decimals to hundredths by reasoning about their size."

The lack of integration across fraction and decimal notation in the intervention literature may be unfortunate. As Hurst and Cordes (2018) demonstrated, although fourth- through seventh-grade typical achievers have greater experience with fraction notation, they are more accurate in processing decimal than fraction notation. Similarly, Hoof, Degrande, Ceulemans, Verschaffel, and Van Dooren (2018) found that typical learners in the upper elementary grades develop an understanding of decimal numbers before achieving adequate understanding of fraction magnitudes.

These and other studies (e.g., DeWolf, Grounds, Bassok, & Holyoak, 2014; Iuculano & Butterworth, 2011) suggest that decimal notation of rational numbers may be more accessible than fraction notation. Moreover, evidence indicates that many students operate as if each rational notation type represents a unique number system, and they experience difficulty converting between notations (Khoury & Zazkis, 1994; O'Connor, 2001). Additional motivation for an integrated approach across fractions and decimals is provided by DeWolf, Bassok, and Holyoak (2015), who identified separable contributions of decimal magnitude and relational understanding of fractions to early success with algebraic expressions. More recently, Resnick, Rinne, Barbieri, and Jordan (2019) found that three increasingly coherent patterns of reasoning on a decimal comparison task midway through fourth grade predicts fraction comparison at the end of fourth grade, while controlling for start-of-year fraction knowledge; it also predicts sixth-grade mathematics achievement, while controlling for whole number and fraction magnitude understanding and demographic and cognitive factors.

Research thus raises the possibility that children may benefit from instruction on decimal notation concurrent with fraction instruction. Such an integrated instructional approach for introducing rational numbers to struggling learners, where fractions and decimals are taught in coordinated fashion, may consolidate understanding on both notation types, with a more robust understanding of rational number than may be provided by a sole focus on fractions. However, at the present time, it is not clear if understanding decimals helps support learning of fractions, and as Tian and Siegler concluded in their 2017 overview, no study has evaluated whether decimal understanding supports later fraction learning.

The purpose of the present study, therefore, was to assess whether fourth-grade magnitude intervention that addresses fractions and decimals in an integrated way provides added value for improving at-risk learners' rational number understanding over intervention focused exclusively on common fractions. We randomly assigned students, each identified with poor whole-number performance, to three conditions: a business-as-usual control group and two variants of the previously validated fourth-grade intervention on fraction magnitude (FM; Fuchs et al., 2016).

One variant included an integrated component on decimal magnitude (FM+DM). The integrated component is new and has not been evaluated in prior research. To reflect the previously validated FM intervention's full focus on fractions and to control for intervention time, the other variant included a fraction applications component (FM+FAPP). All components of this other variant have been tested in earlier studies. Across these FM conditions, the first 6 of 36 lessons were identical. In remaining lessons, variant components comprised the first 7 min of each 35-minute session; the other 28 min were identical across the FM conditions.

We had three hypotheses. First, based on previous RCTs examining the effects of FM intervention, we expected both FM intervention variants to produce superior fraction outcomes compared to control. Our second hypothesis spoke to expected advantages for the integrated fraction-decimal approach over the full fraction focus on decimal understanding. This hypothesis was based on at-risk students' documented struggle with transfer (Haskell, 2001; National Research Council, 2000), such that decimal magnitude understanding is unlikely to improve when intervention focuses solely on fractions.

Our third hypothesis concerned an advantage for the integrated approach over the full fraction intervention on fraction outcomes. Given studies demonstrating that (a) students are more accurate in processing decimal than fraction notation (DeWolf et al., 2014; Hurst & Cordes, 2018; Iuculano & Butterworth, 2011) and (b) typical learners develop understanding of decimal numbers before understanding of fraction magnitude (Hoof et al., 2018), we anticipated that FM+DM, with its integrated focus on fractions and decimals, would enhance fraction (as well as decimal) performance beyond FM+FAPP, with its unitary focus on the more challenging fractions bipartite notation.

An advantage for the integrated focus is also consistent with present understanding about how rational number magnitude representation develops (e.g., Siegler et al., 2011). Children's initial rational number learning relies on rule-based strategies. With opportunities to apply rules across contexts, while integrating rules across problem features, the hope is that reasoning improves, insight deepens, and analog representation grows. Although decimals and fraction share fundamental numerical properties, making them different from and more difficult than whole numbers, decimals and whole numbers share base-10 syntax, which is more familiar than bipartite fraction notation. Learning rule-based strategies for understanding and operating with rational number magnitude within base-10 notation, via decimals, *while* learning how the distinctive fraction notation shares fundamental principles about rational numbers with decimal magnitude, may ease the developmental pathway from rule-based performance to meaningful representation of rational number.

Method

Participants

Participants were at-risk fourth graders in a large U.S. city. Risk was defined as scoring below the 35th percentile on the Wide Range Achievement Test (4th ed.; WRAT-4; Wilkinson & Robertson, 2006), which at this grade level in this population reflects whole-number skill. We stratified selection so approximately half the sample performed below the 15th percentile and half between the 16th and 34th percentiles. Given that this study did not focus on intellectual disability, students were excluded if they performed below the 9th percentile on both subtests of the Wechsler Abbreviated Scales of Intelligence (2nd ed.; WASI; Wechsler, 1999).

Because more students qualified than we had resources to include, we randomly sampled, stratifying by less versus more severe risk status, 240 at-risk students (46% below the 15th percentile; 54% between the 16th and 34th percentile) from 58 classrooms in 12 schools (< 8 students per classroom). We randomly assigned these students to FM+DM, FM+FAPP, and CON, across classrooms, with 80 in each condition. The following number of students left the study before it ended: four from FM+DM because they moved; six from FM+FAPP because they moved, and one who was dropped due to severe behavioral difficulties; and four CON because they moved. The final sample thus included 76 in FM+DM, 73 in FM+FAPP, and 76 in CON.

Screening standard score means for FM+DM, FM+FAPP, and CON, respectively, were 85.39 (*SD* = 8.14), 84.96 (*SD* = 8.41), *M* = 84.72 (*SD* = 6.54) on WRAT-4. Respective scores on WASI Vocabulary for the three conditions were 95.38 (*SD* = 15.98), *M* = 97.12 (*SD* = 13.23), *M* = 94.67 (*SD* = 13.96); on WASI Matrix Reasoning, *M* = 94.38 (*SD* = 13.98), *M* = 96.14 (*SD* = 12.81), *M* = 93.29 (*SD* = 12.76). In the FM+DM, FM+FAPP, and CON conditions, respectively, 42%, 45%, and 46% were male; 38%, 44%, and 46% were African American; 15%, 16%, and 18% white non-Hispanic; 24%, 26%, and 26% white Hispanic; 12%, 16%, and 13% received special education services (67%, 50%, and 60% of whom were classified with a learning disability); 22%, 19%, 20% were English-learners; and 89%, 89%, and 87% qualified for subsidized lunch. ANOVA and chi-square analyses indicated the groups did not differ on any screening or demographic variable and that students who left the study did not differ from students who remained in the study on any screening or demographic variable.

Screening Measures

The following screening measures were administered before pretesting began. With *WRAT-4* (Wilkinson & Robertson, 2006), students solve up to 40 computation problems of progressive difficulty in 10 min. Reliability for this age group is .94. With *WASI Vocabulary*, students identify pictures (4 items) and define words (38 items). Students receive a score of 1 (correct) or 0 (incorrect) on the picture items and can receive a score of 0, 1, or 2 on the word items based on the scoring manual's guide on assessing the sophistication of the students' answer. Testing discontinues after a student earns five consecutive scores of 0. Reliability for this age group is .88. With *WASI Matrix Reasoning*, students solve puzzles by completing a pattern on each page by selecting one of the five choices on the bottom of the page. Each puzzle is increasingly difficult. Testing discontinues after a student makes four consecutive errors or four errors in any five items. Reliability for this age group is .93.

Outcome Measures

Six outcome measures were administered before intervention and after intervention. Two assessed fraction magnitude understanding as addressed in both FM conditions; two assessed fraction applications as addressed in the FM+FAPP condition; and two assessed decimal magnitude understanding (with three resulting scores), as addressed in the FM+DM condition. These measures thus tapped performance at various transfer distances from the intervention's content, and for some measures, transfer distance differed for FM+FAPP versus FM+DM. Figure

1 provides an outline of transfer distance as a function of FM condition. Also, in the following outcome measures section, we provide additional context and explanation (and see table titles for reminders).

Fraction magnitude understanding. We included two measures of fraction magnitude understanding, one near-transfer and one far-transfer with respect to the FM intervention content. The near-transfer measure was the *0-2 Fraction Number Line* task (Hamlett, Schumacher, & Fuchs, 2011, adapted from Siegler et al., 2011). With this computer assessment, students place fractions less than one, equal to one, and greater than one (including mixed numbers) on a number line with endpoints 0 and 2. On each item, students see the number underneath the 0-2 number line (marked only with end-points) and estimate where the fraction goes on the line. The 20 items are: $\frac{2}{3}$, $\frac{7}{9}$, $\frac{5}{6}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{19}$, $\frac{3}{8}$, $\frac{7}{4}$, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{7}{6}$, $\frac{15}{8}$, $1\frac{1}{8}$, $1\frac{1}{5}$, $1\frac{5}{6}$, $1\frac{2}{4}$, $1\frac{11}{12}$, $\frac{5}{5}$, and 1. The score for each item is the absolute difference between where the student estimate the number goes and where it actually goes. Scores are divided by 2 (for the 0-2 number line) and averaged across items to yield the average absolute error (if multiplied by 100, this indicates the average percentage of absolute error [PAE]). Because lower scores indicate greater accuracy, we multiplied scores by –1 for data analysis (higher scores indicate stronger performance). Test-retest reliability is .80.

We considered this computer number line task near transfer for both FM conditions, because during FM intervention, students estimate placement exclusively on paper (not computer) number lines, by writing benchmark numbers and marking areas of greater and lesser magnitude. By contrast, on the unfamiliar computer assessment, students cannot execute written strategies and thus instead engage in a purer form of estimation.

The far-transfer measure for both FM conditions comprises *22 released items from the National Assessment of Education Progress* (NAEP; U.S. Department of Education, 1990-2009). Nineteen items assess fraction magnitude understanding; eight require students to identify fractions and fraction equivalencies with pictures (part-whole understanding), one item is fraction subtraction, and two items assess decimal magnitude understanding. Fourteen items are multiple choice (with four choices each), four are short answer, one requires a drawing or explanation about magnitude, one requires shading a fraction of a picture, and two require placement of on a number line. The maximum score is 27 (two questions have multiple parts, scored separately). Testers read each problem aloud (two times, if needed). $\alpha = .86$.

Fraction applications. *Fraction Calculations*, from the *Fraction Battery*-revised (Schumacher, Namkung, Malone, & Fuchs, 2013) includes *Fraction Addition* with 12 addition problems (five with like denominators, seven with unlike denominators) and *Fraction Subtraction* with 12 subtraction problems (six with like denominator and six with unlike denominators). The score is combined across both subtests (maximum score = 41 points, 24 for correct numerical answers; 16 for correctly reducing answers [not all items need reducing]). $\alpha = .94$. This measure is deemed near-transfer for both FM conditions because fraction addition and subtraction with like denominators is a small component of the FM intervention (3 or 36 lessons). However, note that the FM+FAPP condition received more practice than FM+DM, in the context of solving the addition and subtraction word problems.

Fraction Word Problems, from the *Fraction Battery*-revised (Schumacher et al., 2013), includes 12 word problems: six change-increase word problems (e.g., Today, Sam ate $\frac{3}{8}$ of a pizza for lunch. Then, he ate another $\frac{2}{8}$ of the pizza for dinner. What fraction of the pizza did Sam eat today?) and six change-decrease word problems (e.g., Paul bought $\frac{9}{10}$ pound of jellybeans.

He ate $\frac{3}{10}$ pound of those jelly beans at the movies. How many pounds of jelly beans does Paul have left?). All fractions have the same denominator. Testers read each word problem aloud, twice if requested. Students receive credit for a correct numerical math answer (1 point for correct answer; up to 2 points if correctly reduced on eight of 12 problems) and a correct label. The maximum score is 30 (18 numerical answer points; 12 label points). $\alpha = .78$.

Decimals. The *0-1 Decimal Number Line* task (Malone, Kelley, & Fuchs, 2014, adapted from Siegler et al., 2011) is a computer assessment in which students place tenth and hundredth decimals on a number line labeled only with endpoints. On each item, students see a decimal underneath the 0-1 number line and estimate where the decimal goes. The 10 items are 0.6, 0.95, 0.7, 0.58, 0.9, 0.38, 0.69, 0.4, 0.82, 0.5, 0.75, 0.47, 0.8, 0.3. Like the Fraction Number Line task, the score for each item is the absolute difference between where the student estimates the decimal goes and where it actually goes on the number line. These scores are averaged across items and multiplied by –1 for data analyses so that higher scores indicate stronger performance. Test-retest reliability on a similar number line assessment is .80. We deemed this task near transfer for students in the FM+DM condition (because intervention taught strategies with paper-pencil number lines) but far transfer for students in the FM+FAPP condition.

Decimal Magnitude Assessment (Malone & Fuchs, 2014) includes 22 decimal-magnitude items in four sections. (1) Eight compare items with tenths, hundredths, and thousands require students to write the less than, greater than, or equal sign between two decimals. Two items are near transfer (similar to problems taught in intervention); six are far transfer (dissimilar to problems taught in intervention). (2) Four near transfer ordering items require students to order three decimals (all items include a mix of tenths and hundredths). (3) Three far-transfer items (not addressed in intervention) require students to write a number that comes between two decimals (e.g., .5 and .6. (4) Seven multiple-choice items test knowledge of estimation, place value, and decimal comparisons (e.g., "Circle the number nearest to 0.675: 0.98, 0.5, 0.7, and 700"). Four are near transfer with respect to the content addressed in DM (but far transfer for the FM+FAPP condition). Three are far transfer with respect to the content address in DM and thus far transfer both conditions. Across the four sections, these 22 items create two subtests, referred to as Decimal Magnitude Assessment – DM Similar with 10 items (α = .81) and Decimal Magnitude Assessment – DM Dissimilar with 12 items (α = .86).

The Fraction Magnitude (FM) Component Provided in Both Intervention Conditions

To provide the FM instruction, FM+DM and FM+FAPP both relied on *Fraction Face-Off!*, which is a validated intervention for improving fraction magnitude and applications (see Fuchs, Malone, et al., 2016 for more information; see Fuchs, Schumacher, Malone, & Fuchs, 2015 for a complete manual). The intervention builds fraction magnitude understanding via processing of benchmark fractions (e.g., $\frac{1}{2}$). Activities include comparing, ordering, and placing fractions on 0-1 and 0-2 number lines as well as finding fraction equivalencies for fractions less than, equal to, and greater than 1. Fraction tiles, fraction circles, and number lines are used to represent key ideas. *Fraction Face-Off!* is conducted in dyads, three times per week for 12 weeks; each lesson is approximately 35 min. The intervention also incorporates a small focus (3 of 36 lessons) on fraction addition and subtraction, which includes the concept of fraction quantities increasing and decreasing, equivalent fraction, and procedures for solving number problems and checking the reasonableness of answers.

With *Fraction Face-Off!*, concepts are taught first and then in conjunction with efficient strategies to mirror the concepts and produce correct answers. Sessions are organized in segments, titled to convey the sports theme inherent in the program's title: Warm-Up, Training, Relay, Sprint, and the Individual Contest. The instructional approach is explicit. Warm-up, which is where the DM and FAPP components are addressed, starts on Lesson 7. We explain these components in the next session. In Training, tutors explain and model new ideas and strategies with worked examples and scaffold student understanding and performance as students gradually take responsibility for larger portions of problem solutions. During Relay, tutors provide guided practice as students take turns explaining their thinking as they solve problems; tutors assess and correct misconceptions. To reduce cognitive load in learning new solution strategies, students have access to strategic problem-solving (“help”) cards that outline steps and thought processes for problem solution. As students become fluent with a problem type, the help card is faded. During Sprint (starting in Lesson 10), tutors lead fluency-building activities on key fraction skills, such as generating benchmark equivalent fractions). In the Individual Contest, students complete acquisition and review problems independently followed by corrective feedback. Systematic cumulative review, with interleaved problem types, is incorporated throughout.

Fraction Face-Off! includes a self-regulation system to encourage students to work hard and accurately, listen carefully, and follow directions. Tutors set a timer to beep three times per lesson at random intervals. When the timer beeps, tutors check if both students are on task. If so, they get a checkmark, each worth a “half dollar,” in their “Checkbook.” Students can also earn “half dollars” (and later “quarter dollars”) for accurately completing “bonus problems” on the Individual Contest. To promote hard work on all problems, tutors do not inform students which practice problems are eligible for bonus points until all work is completed. Tutors distribute each student’s earned fraction money at the end of the lesson. On the third lesson each week, students have the opportunity to buy a prize from the “Fraction Store” or save money. Prices are listed in whole-dollar amounts; students convert fraction money to whole dollars, providing additional practice with fraction equivalencies. This self-regulation system was implemented throughout the 35-min sessions (with FM, DM, and FAPP intervention components).

The DM and FAPP Components

The 7-min Warm Up Segment is introduced in Lesson 7. This is where the DM *or* the FAPP component is taught. To ensure that the correct condition is administered to the correct students, worksheets and materials were color-coded.

The DM component. The DM component's topics parallel the methods used with fractions in *Fraction Face-Off!*. Instruction reinforces fraction and decimal magnitude in an integrated way, with number lines and fraction tiles representing decimal-fraction equivalencies. Rewriting decimals to fractions is a major activity used to teach central ideas (Khoury & Zazkis, 1994; O'Connor, 2001), including that fractions can be written as equivalent decimals; digits after the decimal point refer to a part of a whole; the number of digits after the decimal point does not indicate magnitude (Moskal & Magone, 2000); decimals with tenths have a different denominator than decimals with hundredths; and when comparing decimals with tenths and hundredths, tenths must be converted to hundredths. Students convert decimals to fractions with tenths and hundredths when comparing, ordering, number line placement, and finding equivalencies activities.

Lessons 7-12 focus on writing decimal-fraction equivalencies with tenths. Lessons 13-15 focus on comparing decimal tenths to fraction tenths (by rewriting decimals as fractions). After rewriting, students explain how the two fractions have the same denominator and the fraction with "more parts" is the bigger fraction (a comparing strategy in the FM program). Lessons 16-18 focus on placing decimal tenths on the 0-1 number line using the $\frac{1}{2}$ benchmark; Lessons 19-

21, on decimal-fraction equivalencies with hundredths; Lessons 22-30 rely on the same teaching methods, but mixing tenths and hundredths (e.g., 0.6 and 0.53). In Lessons 31-33 students order decimals of tenths and hundredths. Lessons 34-36 are review. For additional information on the DM component, contact the first author.

The FAPP component. The FAPP component is the additive word-problem component previously assessed as a component of *Fraction Face-off!* in Fuchs, Malone et al. (2016), which contrasted the effects of additive versus multiplicative fraction word-problem intervention. See Fuchs, Malone et al. for a full description of the FAPP component.

This FAPP component provides students with instruction and opportunities to apply fraction magnitude judgments by changing fractional values within fraction change-increase and fraction change-decrease word problems. Solving change-increase and decrease word problems also provides students practice with fraction addition and subtraction. With this FAPP condition, students learn to first categorize word problems into problem types (change-increase word problems, change-decrease word problems) and then follow that word-problem type's solution strategy.

Fidelity of Implementation

We conducted frequent live observations and audiotaped all intervention sessions to monitor fidelity of implementation (FOI) and provide ongoing corrective feedback. To quantify FOI, we randomly sampled 46% (1322) of the 2,880 recordings across 80 groups and 36 sessions. Tutor, condition, and group were sampled comparably. Using a FOI checklist, research assistants listened to each recording to assess the extent to which tutors implemented each intervention lesson as intended. Across tutors (n = 20) and both conditions, tutors addressed 96.61% (*SD* = 2.12%) of essential points: 96.70% (*SD* = 2.19%) in FM+DM and 96.52% (*SD* =

2.07%) in FM+FAPP. Tutors addressed 95.84% (*SD* = 4.27%) of the DEC component's items; 94.87% (*SD* =3.86%) of the WP component's items. Using within-tutor analyses, there were no significant differences between conditions for FOI, F (1, 19) = 1.06, p = .42.

Fraction, Decimal, and Word-Problem Instruction Provided by the Schools

To describe the schools' fourth-grade rational number instruction, we relied on two sources: an analysis of the fraction components of the district's fourth-grade math program, enVisionMATH (Scott Foresman-Addison Wesley, 2011), and a questionnaire completed by participating teachers who taught math. Nearly all these teachers had students in the intervention and control conditions.

The district's program. enVisionMATH addresses fractions at fourth grade in two units: Understanding Fractions and Adding/Subtracting Fractions, with 70% of lessons allocated to understanding fractions. For understanding, the program relies mainly on part-whole understanding by using shaded regions and other area model manipulatives, while encouraging students to write and draw when explaining concepts. In a single lesson, benchmark and equivalent fractions address magnitude decisions (number lines are not used). Adding and subtracting fractions are taught via procedural rules. Fraction word problems are addressed dominantly with change and equal sharing word problems, with a smaller emphasis on multiplicative word problems.

Questionnaire. The second source of information was the questionnaire completed by the 39 classroom teachers who taught math in the 58 participating classrooms. (Note that preliminary analyses, which cross-classified teachers and classrooms to reflect 58 classrooms taught by 39 math teachers, estimated variance for teacher at 0, so the teacher random effect was dropped from analyses.) Respondents described the schools' fractions, decimals, and word-problem instruction. Five of the 39 math teachers reported using only the Common Core Math Standards; one only the district's mathematics program, enVisionMATH; and 33 a combination of Common Core, enVisionMATH, and state standards measured on the state's test.

With respect to *fractions*, teachers reported teaching fraction magnitude with the following percentages of instructional time allocated to cross-multiplying (17%), number lines (15%), benchmarking fractions (12%), finding common denominators (22%), drawing pictures (15%), using manipulatives (7%), thinking about the meaning of the numerator and denominator (10%), and other (3%). In terms of fraction calculations, all but one math teacher taught addition and subtraction. Two teachers indicated they did not teach *decimals*. The remaining teachers reported teaching decimal magnitude-assessment strategies in conjunction with place-value charts (50%), base-10 blocks (22%), number lines (21%), place value tiles (5%), and other (2%). They allocated the following emphasis to activities involving place value (22%), comparing (21%), ordering (17%), addition and subtraction (16%), number lines (11%), and money (13%). They reported using the following decimal representations: graphs (97%), number lines (75%), manipulatives (66%), and other (6%). All teachers reported teaching *fraction word problems*, by relying on a variety of methods: writing equations (21%), representing problem narratives with pictures (20%), identifying key words (17%), naming problem types (17%), making tables (10%), and other (2%).

Major Distinctions between Instruction in the Control Group Versus Intervention

Based on the district's math program and teacher reports, we identified the following major distinctions between control versus intervention instruction. With respect to *fractions*, the control group focused more on part-whole while intervention focused more on magnitude understanding. Second, the control group relied more on procedural methods for comparing fractions, whereas intervention focused more on conceptual and magnitude understanding. This included a stronger emphasis on understanding how to compare fraction magnitudes by benchmarking to ½ and less reliance on cross-multiplication of whole numbers to compare fractions. Third, intervention provided a stronger emphasis than the control group on assessing fraction and decimal magnitude with number lines. Fourth, intervention conditions restricted fraction denominators to 12, whereas the control group included denominators to 100.

In terms of *integrating fraction and decimal-magnitude understanding*, this was the emphasis in the FM+DM condition, in which fraction and decimal concepts were taught in coordinated fashion. By contrast, nearly a third of teachers did not allocate any instruction to linking fractions with decimals. When they did, control group instruction typically relied on linking notations via graphical representations, manipulatives, money, and calculators without comparing, ordering, or number line activities. By contrast, FM+DM, which also included manipulatives and real-life applications, placed a stronger emphasis on strategies when comparing, ordering, and making number-line placements.

Finally, control-group *word-problem* instruction focused more on drawing pictures and identifying key words, whereas FM+FAPP taught students to identify word-problem types and required them to rely on problem-model number sentences to structure their solutions to fraction change word problems.

Mathematics Instructional Time for Intervention versus Control Students

On the questionnaire, teachers reported that math was taught in 60-90 min periods five days per week. During intervention, students typically missed the classroom's math instructional time or the school's intervention period. Of the 62% of intervention students who missed classroom math instruction, 79% missed core math instruction, 7% missed math centers, and

14% missed some other type of math instruction, typically spiral math review. Of the 38% of intervention students who did not miss classroom math instruction, 84% instead missed the school's designated intervention block (math or reading), 11% missed part of the reading block, and 5% missed another activity (typically seat work). Nearly half (42%) of students in the intervention conditions also received the school's supplemental math intervention, for an average of 133.71 minutes (*SD* = 51.46) per week; 45% of control students received the school's supplemental math intervention, for an average of 140.29 minutes (*SD* = 65.84) per week. In these ways, students across conditions received similar amounts of math instruction.

Procedure

In August/September, we administered WRAT, NAEP, and Decimal Magnitude Assessment in one whole-class session. In mid-September, students who performed below the 35^{th} percentile on WRAT participated in the two individual testing sessions, including both WASI subtests and both number line tasks. Those met the WASI inclusion criterion participated in one small-group testing session, including Fraction Calculations and Fraction Word Problems. The intervention, which was conducted three lessons per week for 12 weeks, began in late October and continued through the first week of February.

Tutors were employed by the research grant. Most were pursuing a master's degree. Each was responsible for 1-4 groups; all but three had two FM+DM groups and two FM+FAPP groups. Training occurred in two phases. The first phase involved 20 hours of initial training on the manualized intervention, when tutors were familiarized with the intervention procedures and practiced delivering lessons with peers. Although the program is scripted, tutors practiced delivering content without reading scripts, which are provided only to guide implementers with a concrete representation of how the session and explanations are designed to occur. All tutors achieved 95% implementation accuracy before working with children. The second phase of training included weekly meetings, in which additional training and support for teaching content in upcoming weeks was provided.

Following intervention, schools closed for approximately two weeks due to severe weather. Therefore, prior to posttesting, we administered one intervention booster session, which included 17 review problems (the booster session did not include DECM or FAPP content). In March, we re-administered the NAEP and Decimal Magnitude Assessment in one whole-class session, the computer number line tasks and Fraction Word Problems in one individual session, and Fraction Calculations in one small-group session.

Data Analysis and Results

Students (level 1 units) were partially nested and cross classified in small groups (level 2[a] unit, occurring only in the FM+DM and FM+FAPP intervention arms) and classrooms (level 2[b] unit). Partial-nesting analyses followed Bauer, Sterba and Hallfors (2008) and Sterba (2015), whereby a random effect for nesting at the small-group level is employed in each intervention arm, but not the CON arm. This procedure involves estimating a random effect that is toggled into the model for each intervention arm and toggled out of the model for the CON arm. Also, residual (person-level) variance was allowed to differ across study arms, to avoid the requirement that the CON arm necessarily have a smaller model-implied variance than the intervention arms (Bauer et al., 2008; Sterba et al., 2014). This basic partial nesting multilevel model was expanded to account for the cross-classification by also estimating a random intercept at the classroom-level. Intra-class correlation coefficients (ICCs; not controlling for pretest) were computed taking the cross-classification into account. Accordingly, classroom ICCs are:

$$\mathrm{ICC}_{cl|trt} = \frac{\hat{\tau}^{cl}}{\hat{\tau}^{cl} + \hat{\tau}^{sg} + \hat{\sigma}^{2(trt)}} \quad \text{and} \quad \mathrm{ICC}_{cl|cont} = \frac{\hat{\tau}^{cl}}{\hat{\tau}^{cl} + \hat{\sigma}^{2(cont)}}$$

where $\hat{\tau}^{cl}$ is the random intercept variance at the classroom level and $\hat{\tau}^{sg}$ is the random effect variance for a given intervention arm (FM+DM or FM+FAPP) at the small-group level. $\hat{\sigma}^{2(trt)}$ is the person-level residual variance in a given intervention arm (FM+DM or FM+FAPP) and $\hat{\sigma}^{2(cont)}$ is the person-level residual variance in the CON arm. The small-group ICC in a given intervention arm is:

$$\text{ICC}_{sg|trt} = \frac{\hat{\tau}^{sg}}{\hat{\tau}^{cl} + \hat{\tau}^{sg} + \hat{\sigma}^{2(trt)}}$$

Cross-classified partially nested multilevel models were run in SAS Proc Mixed using restricted maximum likelihood estimation (REML) to test differences between FM+DM versus CON, FM+FAPP versus CON, and FM+DM versus FM+FAPP. Standard errors were corrected for small cluster size bias as described in Kenward and Rogers (1997). Degrees of freedom for *t*-tests of fixed effects were approximated using the procedure of Kenward and Rogers (1997) (ddfm=kr option in SAS) because they do not have a known reference distribution for complex variance component structures such as the one fitted here (Bauer et al., 2008). We calculated ESs for partial nesting designs as an across-arm conditional absolute mean difference (controlling for pretest) divided by the residual variance within the CON arm only (Hedges & Citkowicz, 2014; Sterba, 2017). This implies that intervention effects are measured in terms of within-classroom *SD*s as computed under no manipulation, after controlling for pretest.

Table 1 includes pretest/posttest means and *SD*s and Table 2 shows ICCs for each outcome. Tables 3-6 show results of cross-classified partially nested multilevel models for the outcomes (Models 1-7). ESs reported in the far-right column of Tables 3-6. Students in both FM intervention conditions outperformed CON students on both fraction magnitude measures (0-2 Fraction Number Line, which was near-transfer for both intervention conditions, and NAEP, which was far-transfer for both intervention conditions); on both fraction application measures (Fraction Calculations, which was near-transfer for both intervention conditions, and Fraction Word Problems, which was near-transfer for FM+FAPP but far-transfer for FM+DM); and on one decimal outcome (0-1 Decimal Number Line, which was far-transfer for FM+FAPP but near-transfer for FM+DM). On the Decimal Magnitude Assessment – Near Transfer subtest (near-transfer for FM+DM but far-transfer for FM+FAPP), FM+DM outperformed FM+FAPP and CON students. On the Decimal Magnitude Assessment – Far Transfer subtest (far-transfer for both intervention conditions), there were no significant effects.

Discussion

The main purpose of the present study was to assess whether fourth-grade magnitude intervention that addresses fractions and decimals in an integrated way provides added value for improving at-risk learners' rational number performance beyond what accrues with intervention focused exclusively on common fractions. We randomly assigned students with low whole-number skill to a control group and two variants of a previously validated fourth-grade FM intervention. FM+DM included a new integrated component on decimal magnitude. To reflect the validated FM intervention's full focus on fractions and to control for intervention time, FM+FAPP included a fractions applications component. In this discussion, we refer to the FM+DM as *integrated fraction-decimal intervention* and refer to FM+FAPP as *full fraction intervention*. We discuss findings in terms of the study's three hypotheses.

Does Each Variant of Fraction Magnitude Intervention Improve Fraction Performance over the Control Group?

Based on robust effects on fraction outcomes in previous RCTs examining the effects of the FM intervention, *Fraction Face-Off!* (Fuchs, Malone et al., 2016; Fuchs, Schumacher et al., 2013, 2014, 2016), our first hypothesis was the full fraction intervention would produce superior fraction outcomes compared to control. Accordingly, on both fraction magnitude outcomes, full fraction intervention outperformed the control group. The ES on the near-transfer fraction number line task was 1.07; on far-transfer NAEP, 0.59. On fraction applications, ESs were 0.82 on word problems and 3.14 on calculations. Therefore, across the four fraction outcomes, the mean ES was 1.41. The present study thus provides additional replication of FM intervention's efficacy for improving fraction performance.

We also hypothesized positive effects on fraction outcomes for FM intervention that incorporates integrated fraction-decimal instruction. We anticipated this even though this variant is less closely aligned with the previously validated FM intervention than the full fraction intervention just discussed. That is, over the 12 weeks of integrated intervention, 210 min of fraction instruction (7 min of 30 lessons: 7 - 36) were diverted from contextualizing and applying magnitude understanding exclusively in the context of fractions (as in the full fraction condition) to teaching students about connections between decimals and fractions. This redirection and the demands involved in processing a second form rational number runs the risk of increasing the load on at-risk learners' cognition. Even so, the integrated fraction-decimal intervention also produced significantly stronger performance compared to control on all four fraction outcomes. Moreover, this condition's performance was comparable to that of students in the full fraction intervention on all four fraction outcomes.

In terms of the magnitude of effects, on fraction number line, the study's near-transfer fraction magnitude assessment for both FM variants, the ES was 1.10, the same as the ES for the full fraction intervention (1.07). This suggests no detriment to or confusion about fraction magnitude understanding as a function of adding a component that forges connections between the two rational number systems. Yet, on the other three outcomes, although effects were statistically comparable, ESs were somewhat smaller for integrated fraction-decimal intervention than for full fraction intervention. On far-transfer magnitude understanding (NAEP), the ES for integrated fraction-decimal intervention versus control was 0.36, compared to 0.59 for full fraction intervention (despite that NAEP included two decimal magnitude items). The somewhat smaller ES for integrated intervention was surprising because, despite the potentially heavier cognitive load, we expected integration across fractions and decimals to enhance fraction understanding, a point we return to later.

On word problems, the ES for integrated intervention versus control was 0.63, compared to 0.82 for the full fraction intervention. The ES of 0.63 seems impressive, given that the integrated condition had no instructional focus on word problems. Yet, prior research shows that *additive* fraction word-problem skill improves as fraction magnitude understanding grows, even without an explicit intervention focus on additive word problems (Fuchs et al., 2013). (Note, however, that prior work does indicate the need for explicit word-problem intervention to improve *multiplicative* word problems [Fuchs, Malone et al., 2016], the type of word problem included in the *Fraction Face-Off*! When used in actual practice in schools.) A similar pattern occurred for calculations, with an ES of 2.63 for integrated fraction-decimal intervention over control, compared to 3.41 for full fraction intervention. The ES of 2.63 is also not surprising given prior work showing that fraction addition and subtraction improve as fraction magnitude understanding grows (Fuchs et al., 2013, 2014).

In any case, across the four fraction outcomes, the overall ES for the integrated fraction-decimal variant was 1.23; 1.41 for the full fraction intervention. Thus, ESs were large for both intervention conditions, and with respect to this study's first hypothesis, we conclude that in line with previous RCTs examining the effects of FM intervention (Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013, 2014, 2016), both of this study's variants of the FM intervention produce superior fraction outcomes compared to control.

Does Integrated Fraction-Decimal Intervention Improve Decimal Performance More Than Full Fractions Intervention?

Our second hypothesis anticipated advantages for the integrated fraction-decimal approach over the full fraction focus on decimal understanding. This was based on at-risk students' documented struggle with transfer (Haskell, 2001; National Research Council, 2000), which reflects a low likelihood that decimal magnitude understanding improves in response to an intervention exclusively focused on fractions.

The DM-similar subtest of the Decimal Magnitude Assessment indexed whether students learned the types of decimal and decimal-fraction bridging ideas and tasks taught within the integrated condition. Results on this measure suggested they did. The integrated intervention outperformed the control group, with a large ES of 2.09, and the integrated condition also outperformed the full fraction intervention condition again with a large ES (1.79).

Yet, results on the computer decimal number line task, also a near-transfer task for the integrated fraction-decimal condition, diverged from the hypothesized pattern. That is, although integrated fraction-decimal intervention students outperformed the control group (ES = 0.50), so did full fraction intervention students, for whom the decimal number line task required far transfer. On the positive side, the demonstration of far transfer in the full fraction intervention condition is encouraging. It extends prior work on the FM intervention by showing that FM intervention produces transfer to decimal magnitude understanding.

At the same time, the lack of superior decimal number line task for the integrated intervention over the full-fraction intervention is disappointing. The disparate findings for the near-transfer Decimal Magnitude Assessment – Similar Subtest versus the decimal number line task for the integrated intervention is likely explained by transfer distance. Although both measures were deemed near transfer for the integrated condition, the Decimal Magnitude Assessment's DM - Similar Subtest tasks paralleled items taught during intervention. By contrast, the decimal number line task invoked a purer form of estimation than was required during the intervention's paper-pencil number line activities (see explanation in measures section).

In the context of the present study, the observed pattern of effects raises questions about whether the integrated intervention's effect on the decimal number line measure can be attributed to the fraction-decimal component of the intervention. Instead, comparable performance between the full fractions intervention, with*out* a decimal emphasis, and the integrated intervention, *with* a decimal emphasis, suggests that the fraction-decimal intervention's effect on decimal number line performance may be attributed to the intervention's fraction instruction.

Adding to the question about the integrated fraction-decimal component's efficacy is the lack of significant effects among conditions on the DM - Dissimilar Subtest of the Decimal Magnitude Assessment, a far-transfer task for both FM conditions. ESs were 0.09 for integrated intervention versus control; 0.24 for full intervention versus control. The integrated condition tested in the present study allocated 210 min over the course of 12 weeks to connections between fraction and decimals. Our findings suggest that a stronger emphasis on decimals and their connections with fractions, with broader content coverage and a greater variety of tasks, may be required to deepen understanding.

Does Integrated Fraction-Decimal Intervention Improve Performance More Than Full Fraction Intervention on Fractions or Rational Numbers More Broadly?

This study's final hypothesis was that integrated fraction-decimal intervention would enhance performance more than full fraction intervention on fractions (rational numbers more broadly). As already discussed, we did not find support for stronger effects on fraction intervention with an integrated intervention than for the full fraction intervention. Both conditions outperformed the control group, and the two intervention groups performed comparably. In fact, across the four fraction outcomes, the mean ES for the integrated intervention was strong (1.21), but the ES for the full fraction intervention was somewhat higher (1.41).

The pattern, however, differs when we consider rational number performance more broadly, including fractions and decimals together. Across the full set of seven rational number outcomes, the mean ES for the integrated fraction-decimal condition was 1.06; for the full fraction intervention, 0.94. On the full set of five rational number magnitude understanding tasks, the mean ES for the integrated fraction-decimal condition was 0.82; for the full fraction intervention, 0.52. This suggests an affirmative answer to this study's question, Does an integrated focus on fractions and decimals improve at-risk students' rational number performance? Unfortunately, this advantage for the integrated intervention was driven entirely by students' superior learning (with very large effects) on a near-transfer measure aligned with tasks taught in the integrated intervention (Decimal Magnitude Assessment - DM Similar Subtest). It was not driven by superior fraction magnitude understanding of decimals or fractions and not driven by superior fraction applications.

Finding that integrated fraction-decimal intervention, which required students to conceptualize and transcribe across fraction and decimal notations, did not carry a clear advantage over fractions intervention alone, runs contrary to suggestions in the literature that learning may be enhanced when fractions and decimals are taught in coordinated fashion. Our findings do not support this possibility perhaps because transcribing between the two notations creates cognitive load that interferes with decimal learning for at-risk fourth graders, who have little prior experience with rational numbers along with a history of whole-number mathematics difficulty. Even so, given studies demonstrating that students are more accurate in processing decimal than fraction notation (DeWolf et al., 2014; Hurst & Cordes, 2018; Iuculano & Butterworth, 2011) and considering that typical learners develop an understanding of decimal numbers before achieving adequate understanding of fraction magnitude (Hoof et al., 2018), further investigation appears warranted.

Future research may include studies that test the effects of stronger or broader versions of integrated fraction-decimal intervention or ones with deeper and broader decimal content. This might include for example more time and practice allocated to decimals (after all, the decimal component comprised only 7 of the 35 min within 30 of the 36 sessions). It might also include a greater variety of decimals presented in a greater variety of contexts. Alternatively, as suggested by Moss and Case (1999), it is possible that a strong emphasis on decimals at a younger grade (e.g., grade 2), with linkages between decimal and fraction notation introduced in a subsequent grade (e.g., grade 3) and consolidated during the following year (e.g., grade 4) would represent a more successful strategy for realizing the synergistic effects potentially offered via a coordinated instructional focus across fractions and decimals.

Conclusions

Results provide the basis for three conclusions. First, FM intervention improves at-risk fourth-graders' fraction magnitude understanding and applications, with large effect sizes. Second, effects of FM intervention transfer to decimal number line performance, even without an intervention focus on decimals. Third, an intervention component designed to integrate fraction and decimal magnitude does not provide added value on fraction or decimal performance, except on decimal tasks closely paralleling those practiced during intervention. With respect to this last conclusion, additional research is needed, testing effects of deeper or more comprehensive intervention for integrating fractions with decimals. Research may also assess the value of introducing decimals, potentially the more accessible of the two rational number notations, at an earlier grade to build a stronger platform for later fractions instruction. A staged approach may help at-risk learners capitalize on connections between foundational decimal understanding and more challenging fraction ideas and applications.

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Table 1

Means and Standard Deviations for all Measures

	FM+DM (*n* = 76)		FM+FAPP (*n* = 73)		CON (*n* = 76)	
Measure	Pre *M* (*SD*)	Post *M* (*SD*)	Pre *M* (*SD*)	Post *M* (*SD*)	Pre *M* (*SD*)	Post *M* (*SD*)
Fraction Magnitude						
Fraction Number Line[a]	0.60 (0.12)	0.37 (0.17)	0.57 (0.15)	0.36 (0.19)	0.62 (0.16)	0.52 (0.15)
NAEP (27)	9.95 (3.45)	15.20 (4.32)	9.84 (3.90)	16.03 (4.16)	8.96 (4.07)	13.29 (4.50)
Fraction Applications						
Fraction Calculations (41)	3.46 (4.35)	18.54 (7.94)	4.64 (4.14)	20.85 (6.58)	3.12 (4.06)	8.45 (4.06)
Fraction Word Problems (30)	5.63 (4.60)	10.97 (3.47)	6.67 (4.00)	12.08 (4.02)	5.22 (4.22)	8.79 (3.70)
Decimal Magnitude						
Decimal Number Line[a]	0.30 (0.08)	0.26 (0.09)	0.33 (0.08)	0.28 (0.09)	0.31 (0.08)	0.31 (0.08)
Decimal MA: DM Similar (10)	1.03 (0.78)	5.15 (2.35)	1.14 (1.02)	2.48 (2.26)	1.28 (0.91)	1.94 (1.98)
Decimal MA: DM Dissiminar (12)	1.49 (1.06)	2.76 (2.24)	1.55 (1.22)	2.89 (2.15)	1.92 (1.37)	2.38 (1.91)

Note: CON = control; MA = magnitude assessment; FM+DM = fraction magnitude and decimal magnitude intervention; FM+FAPP = fraction magnitude and fraction applications intervention. Numbers in parentheses in the first column indicate the maximum score for that measure. [a]For the number line measures, scores indicate absolute error (PAE). Note that because lower scores indicate greater accuracy on the number line measures, we multiplied scores by –1 for data analysis (higher scores indicate stronger performance).

Table 2

Classroom and Small-Group (Intervention Only) ICCs across Measures

Measure	Classroom ICC: CON	Classroom ICC: FM+DM	Classroom ICC: FM+FAPP	Small-Group ICC: FM+DM	Small-Group ICC: FM+FAPP
Fraction Magnitude					
Fraction Number Line[a]	.00	.00	.00	.07	.00
NAEP	.04	.04	.04	.00	.06
Fraction Applications					
Fraction Calculations	.04	.10	.03	.06	.00
Fraction Word Problems	.00	.00	.00	.00	.00
Decimal Magnitude					
Decimal Number Line[a]	.00	.00	.00	.00	.05
Decimal Magnitude Assessment: DM Similar	.14	.17	.09	.42	.00
Decimal Magnitude Assessment: DM Dissiminar	.14	.16	.10	.00	.00

Note: All reported ICCs are for posttest (not controlling for pretest). CON = control; FM+DM = fraction magnitude and decimal magnitude intervention; FM+FAPP = fraction magnitude and fraction applications intervention.

Table 3

Fraction Magnitude Cross-Classified Multilevel Model 1, Fraction Number Line (near-transfer for FAP and DM) and Model 2, NAEP (far-transfer for FAP and DM)

Model/Parameter	Estimate	SE	*t*(*df*)‡/ *z*-value*	*p*-value	ES
1. Fraction Number Line[a]					
Fixed effects					
Adj† mean, CON	–0.516	0.016	–32.69 (73)	<.0001	
Adj† mean, FAP	–0.372	0.021	–17.68 (74)	<.0001	
Adj† mean, DM	–0.368	0.020	–18.71 (36)	<.0001	
Adj mean difference, FAP vs. CON	0.144	0.026	5.46 (136)	<.0001	1.073
Adj mean difference, DM vs. CON	0.148	0.025	5.88 (80)	<.0001	1.103
Adj mean difference, DM vs. FAP	0.004	0.029	0.14 (100)	.885	0.030
Pretest effect	0.425	0.073	–5.83 (194)	<.0001	
Random Effects					
Classroom-level variance	0.000	--	--	--	
Group-level variance, FAP arm	0.000	--	--	--	
Group-level variance, DM arm	0.005	0.004	1.10	.135	
Person-level residual variance, CON	0.018	0.003	6.03	<.0001	
Person-level residual variance, FAP	0.033	0.005	6.04	<.0001	
Person-level residual variance, DM	0.020	0.005	4.25	<.0001	
2. NAEP					
Fixed effects					
Adj† mean, CON	13.771	0.392	35.15 (73)	<.0001	
Adj† mean, FAP	15.744	0.429	36.69 (38)	<.0001	
Adj† mean, DM	14.991	0.441	34.02 (76)	<.0001	
Adj mean difference, APP vs. CON	1.973	0.582	3.39 (94)	.001	0.587
Adj mean difference, DM vs. CON	1.221	0.591	2.07 (148)	.041	0.364
Adj mean difference, DM vs. FAP	–0.752	0.615	–1.22 (103)	.224	–0.224
Pretest effect	0.661	0.065	10.13 (209)	<.0001	
Random Effects					
Classroom-level variance	0.000	--	--	--	
Group-level variance, FAP arm	0.702	2.146	0.33	.372	
Group-level variance, DM arm	0.000	--	--	--	
Person-level residual variance, CON	11.282	1.874	6.02	<.0001	
Person-level residual variance, FAP	12.213	2.850	4.29	<.0001	
Person-level residual variance, DM	14.918	2.426	6.15	<.0001	

Note: Adj = adjusted. CON = control; DM = fraction magnitude and decimal magnitude intervention; FAP = fraction magnitude and fraction applications intervention. ES = effect size. †These means are adjusted for pretest and are interpretable where pre-test=0 (i.e. at the overall, across-arm, mean of pretest). ‡*df* for *t*-tests of fixed effects are computed using the Kenward and Rogers (1997) method. **z*-tests of random effects are conservative. [a]Note that because lower scores indicate greater accuracy on 0-2 Fraction Number Line, we multiplied scores by –1 for data analysis (higher scores indicate stronger performance).

Table 4

Fraction Applications Cross-Classified Multilevel Model 3, Fraction Calculations (near-transfer for FAP and DM) and Model 4, Fraction Word Problems (near-transfer for FAP; far-transfer for DM)

Model/Parameter	Estimate	SE	*t*(*df*)‡/*z*-value*	*p*-value	ES
3. Fraction Calculations					
Fixed effects					
Adj† mean, CON	8.688	0.507	17.15 (51)	<.0001	
Adj† mean, APP	20.412	0.833	24.51 (74)	<.0001	
Adj† mean, DEC	18.515	0.922	20.07 (39)	<.0001	
Adj mean difference, FAP vs. CON	11.725	0.917	12.79 (111)	<.0001	3.143
Adj mean difference, DM vs. CON	9.828	0.996	9.87 (50)	<.0001	2.634
Adj mean difference, DM vs. FAP	–1.897	1.204	–1.58 (86)	.119	–0.509
Pretest effect	.322	0.092	3.50 (161)	.001	
Random Effects					
Classroom-level variance	1.920	1.712	1.12	.131	
Group-level variance, FAP arm	0	--	--	--	
Group-level variance, DM arm	3.891	9.534	0.41	.342	
Person-level residual variance, CON	13.917	2.694	5.17	<.0001	
Person-level residual variance, FAP	45.545	7.769	5.86	<.0001	
Person-level residual variance, DM	52.457	12.002	4.37	<.0001	
4. Fraction Word Problems					
Fixed effects					
Adj† mean, CON	8.974	0.382	23.50 (72)	<.0001	
Adj† mean, FAP	11.669	0.466	25.03 (74)	<.0001	
Adj† mean, DM	11.044	0.393	28.11 (73)	<.0001	
Adj mean difference, FAP vs. CON	2.695	0.605	4.45 (140)	<.0001	0.823
Adj mean difference, DM vs. CON	2.070	0.547	3.78 (148)	.0002	0.632
Adj mean difference, DM vs. FAP	–0.625	0.611	–1.02 (144)	.308	–0.191
Pretest effect	0.313	0.056	5.62 (210)	<.0001	
Random Effects					
Classroom-level variance	0.000	--	--	--	
Group-level variance, FAP arm	0.000	--	--	--	
Group-level variance, DM arm	0.000	--	--	--	
Person-level residual variance, CON	10.716	1.798	5.96	<.0001	
Person-level residual variance, FAP	15.940	2.641	6.04	<.0001	
Person-level residual variance, DM	11.876	1.963	6.05	<.0001	

Note: Adj = adjusted. CON = control; DM = fraction magnitude and decimal magnitude intervention; FAP = fraction magnitude and fraction applications intervention. ES = effect size. †These means are adjusted for pretest and are interpretable where pre-test=0 (i.e. at the overall, across-arm, mean of pretest). ‡*df* for *t*-tests of fixed effects are computed using the Kenward and Rogers (1997) method. *z-tests of random effects are conservative.

Table 5

Decimal Magnitude Cross-Classified Multilevel Model 5, Decimal Number Line (far-transfer for FAP; near-transfer for DM)

Model/Parameter	Estimate	SE	*t*(*df*)‡/ *z*-value*	*p*-value	ES
5. Decimal Number Line[a]					
Fixed effects					
Adj† mean, CON	–0.308	0.010	–31.71 (68)	<.0001	
Adj† mean, FAP	–0.273	0.011	–24.95 (39)	<.0001	
Adj† mean, DM	–0.266	0.010	–26.57 (65)	<.0001	
Adj mean difference, FAP vs. CON	0.034	0.014	2.42 (81)	.018	0.406
Adj mean difference, DM vs. CON	0.042	0.014	3.09 (147)	.002	0.502
Adj mean difference, DM vs. FAP	0.008	0.015	0.53 (93)	.601	0.096
Pretest effect	0.301	0.072	–4.20 (221)	<.0001	
Random effects					
Classroom-level variance	0.000	0.000	0.40	.345	
Group-level variance, FAP arm	0.001	0.001	0.41	.340	
Group-level variance, DM arm	0.000	--	--	--	
Person-level residual variance, CON	0.007	0.001	5.83	<.0001	
Person-level residual variance, FAP	0.007	0.002	4.28	<.0001	
Person-level residual variance, DM	0.007	0.001	6.08	<.0001	

Notes. Adj = adjusted. CON = control; DM = fraction magnitude and decimal magnitude intervention; FAP = fraction magnitude and fraction applications intervention. ES = effect size. † These means are adjusted for pretest and are interpretable where pre-test=0 (i.e. at the overall, across-arm, mean of pretest). ‡*df* for *t*-tests of fixed effects are computed using the Kenward and Rogers (1997) method. *z-tests of random effects are conservative. . [a]Note that because lower scores indicate greater accuracy on Decimal Number Line, we multiplied scores by –1 for data analysis (higher scores indicate stronger performance).

Table 6

Cross-Classified Multilevel Model 6, Decimal Magnitude Assessment: DM Similar (far-transfer for FAP; near-transfer for DM) and Model 7, Decimal Magnitude Assessment: DM Dissimilar (far-transfer for FAP and DM)

Model/Parameter	Estimate	SE	*t*(*df*)‡/ *z*-value*	*p*-value	ES
6. Decimal Magnitude Assessment: DM Similar					
Fixed effects					
Adj† mean, CON	0.185	0.032	5.87 (65)	<.0001	
Adj† mean, FAP	0.252	0.034	7.40 (85)	<.0001	
Adj† mean, DM	0.653	0.048	13.51 (43)	<.0001	
Adj mean difference, FAP vs. CON	0.067	0.040	1.70 (126)	.092	0.300
Adj mean difference, DM vs. CON	0.468	0.053	8.90 (62)	<.0001	2.093
Adj mean difference, DM vs. FAP	0.401	0.055	7.32 (70)	<.0001	1.793
Pretest effect	0.143	0.172	0.83 (192)	.408	
Random effects					
Classroom-level variance	0.011	0.005	2.17	.015	
Group-level variance, FAP arm	0.000	--	--	--	
Group-level variance, DM arm	0.050	0.020	2.55	.005	
Person-level residual variance, CON	0.050	0.009	5.61	<.0001	
Person-level residual variance, FAP	0.061	0.012	5.23	<.0001	
Person-level residual variance, DM	0.056	0.013	4.40	<.0001	
7. Decimal Magnitude Assessment: DM Dissimilar					
Fixed effects					
Adj† mean, CON	0.193	0.018	10.85 (75)	<.0001	
Adj† mean, FAP	0.223	0.018	12.23 (83)	<.0001	
Adj† mean, DM	0.204	0.021	9.61 (79)	<.0001	
Adj mean difference, FAP vs. CON	0.030	0.022	1.38 (128)	.171	0.237
Adj mean difference, DM vs. CON	0.011	0.025	0.45 (140)	.657	0.087
Adj mean difference, DM vs. FAP	–0.019	0.025	–0.77 (144)	.441	–0.150
Pretest effect	0.286	0.116	2.47 (200)	.014	
Random effects					
Classroom-level variance	0.003	0.001	2.21	.014	
Group-level variance, FAP arm	0.000	--	--	--	
Group-level variance, DM arm	0.000	--	--	--	
Person-level residual variance, CON	0.016	0.003	5.58	<.0001	
Person-level residual variance, FAP	0.017	0.003	5.38	<.0001	
Person-level residual variance, DM	0.027	0.005	5.89	<.0001	

Notes: Adj = adjusted. CON = control; DM = fraction magnitude and decimal magnitude intervention; FAP = fraction magnitude and fraction applications intervention. ES = effect size. †These means are adjusted for pre-test and are interpretable where pretest = 0 (i.e. at the overall, across-arm, mean of pretest). ‡*df* for *t*-tests of fixed effects are computed using the Kenward and Rogers (1997) method. *z-tests of random effects are conservative.

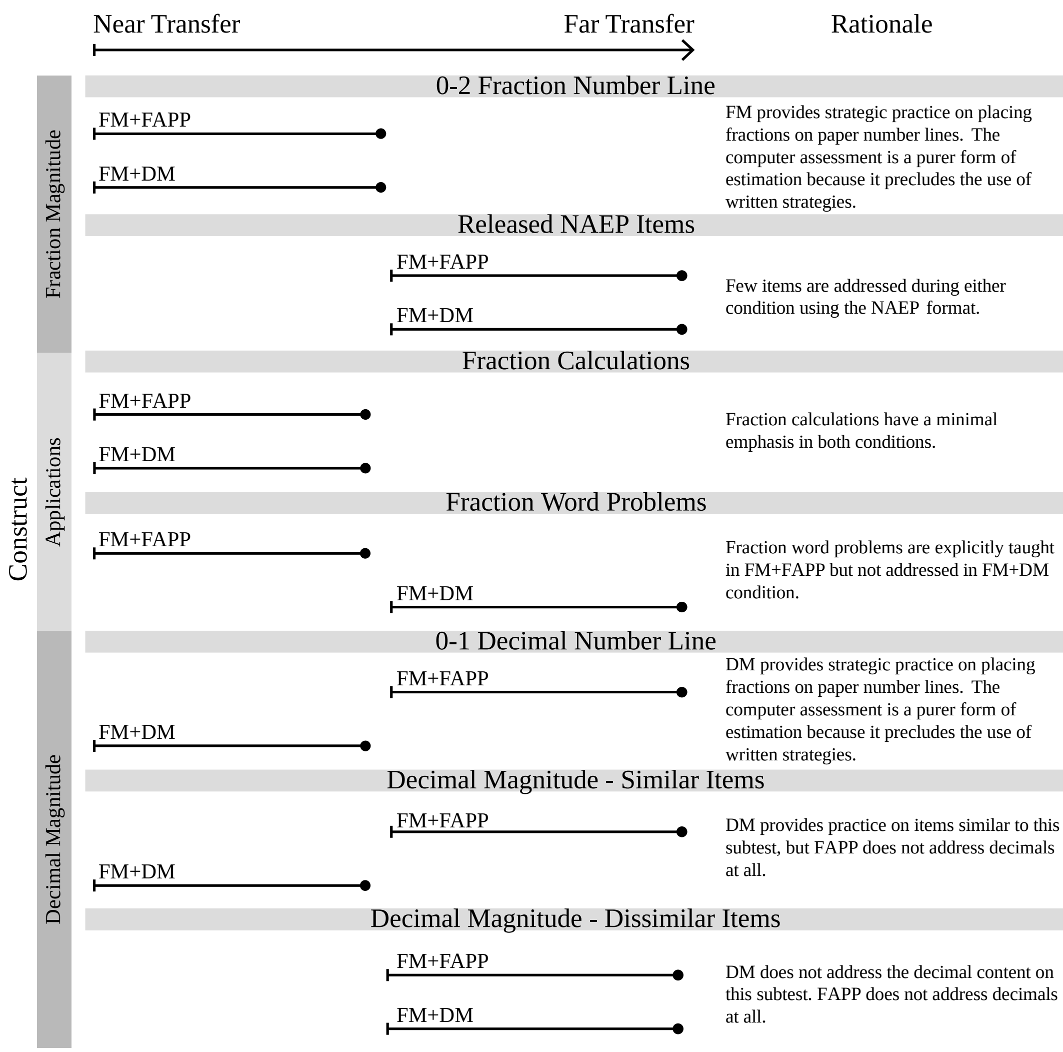

Figure 1. Outline of transfer distance as a function of FM condition.